Direct Forcing Immersed Boundary (DFIB) Method for Mixed Heat Transfer

Dedy Zulhidayat Noor[a]

Ming-Jyh Chern (陳明志)[b]

Tzyy-Leng Horng (洪子倫)[c]

[a]Department of Mechanical Engineering, Institut Teknologi Sepuluh Nopember, Indonesia

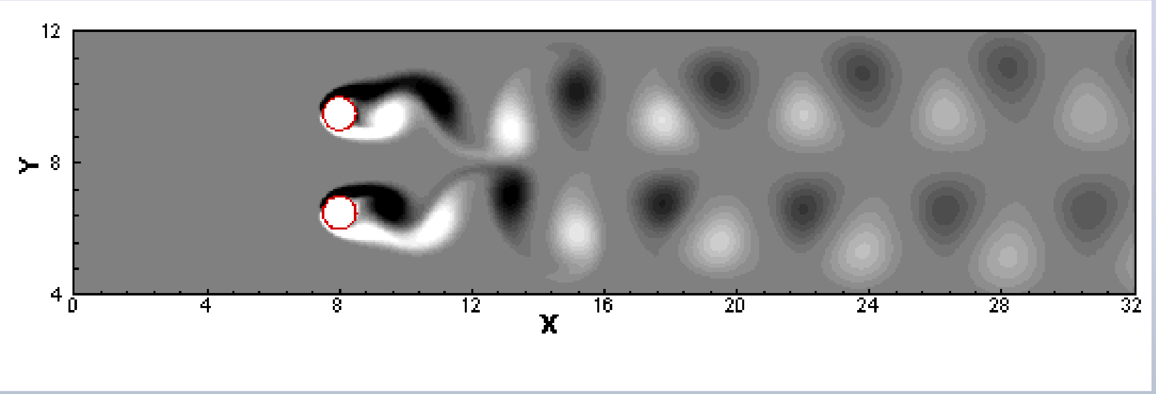
[b]Department of Mechanical Engineering, National Taiwan University of Science and Technology, Taiwan

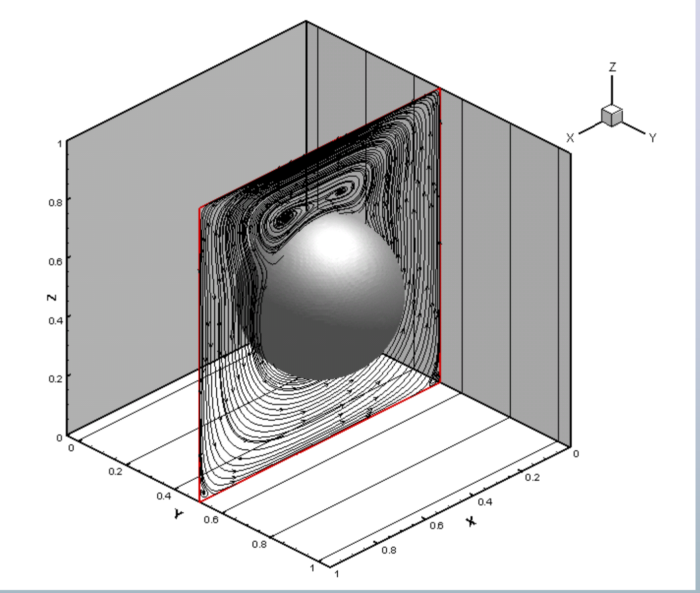
[c]Department of Applied Mathematics, Feng Chia University, Taiwan

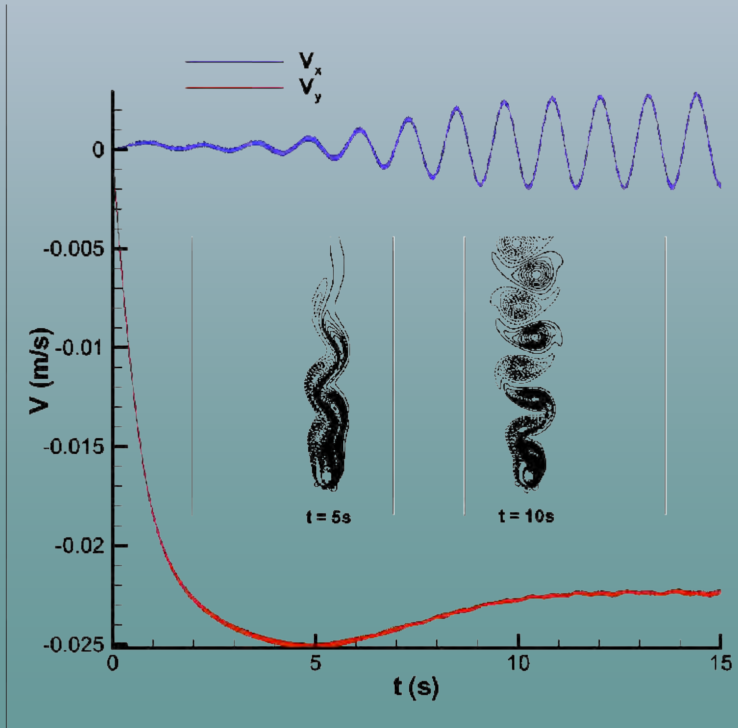
Happy birthday to Tony, my upper classman (學長)!

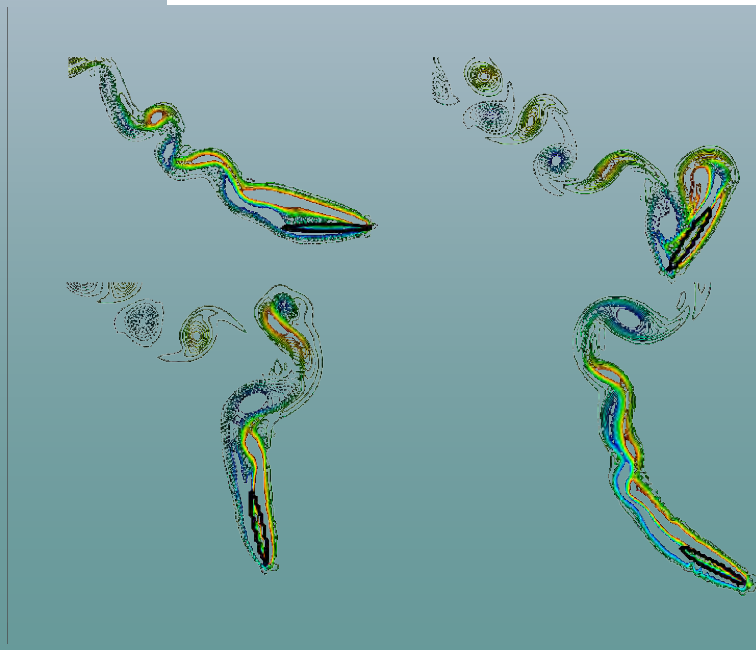
You may wonder why I call him upper class man? I have to because we went to same

We have used DFIB to do









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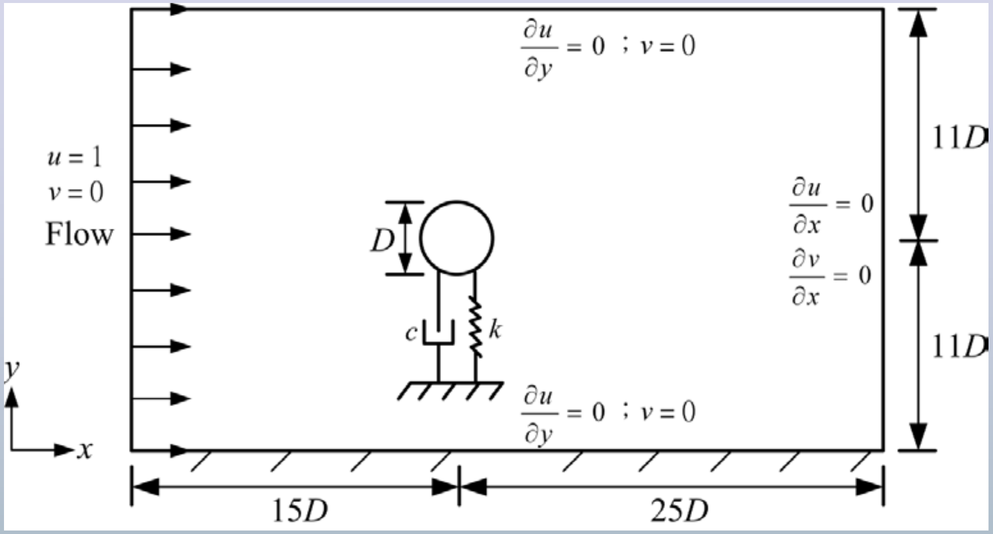

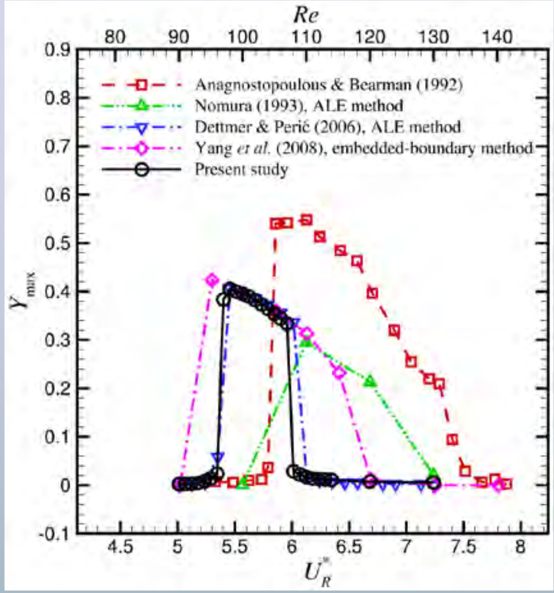

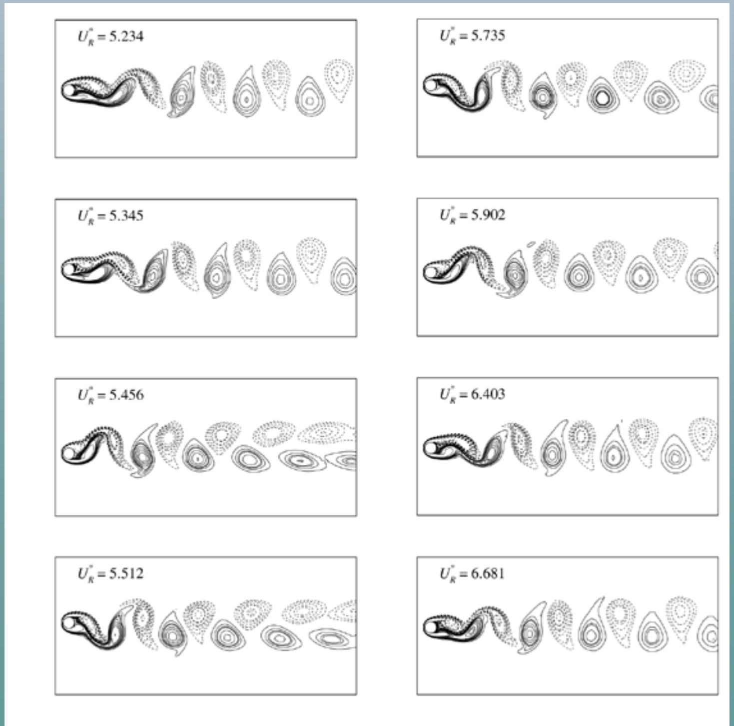

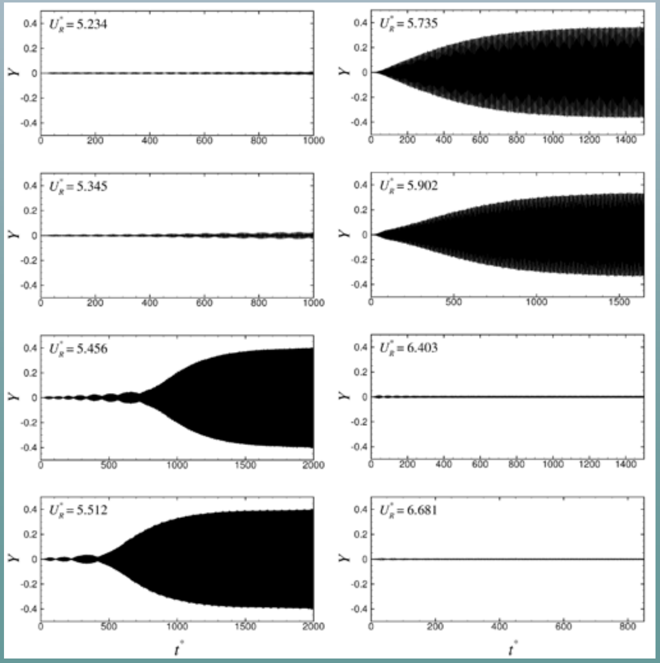

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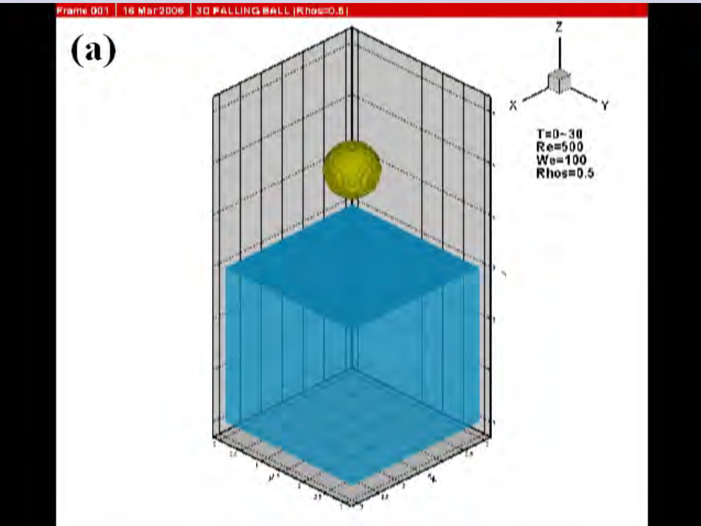
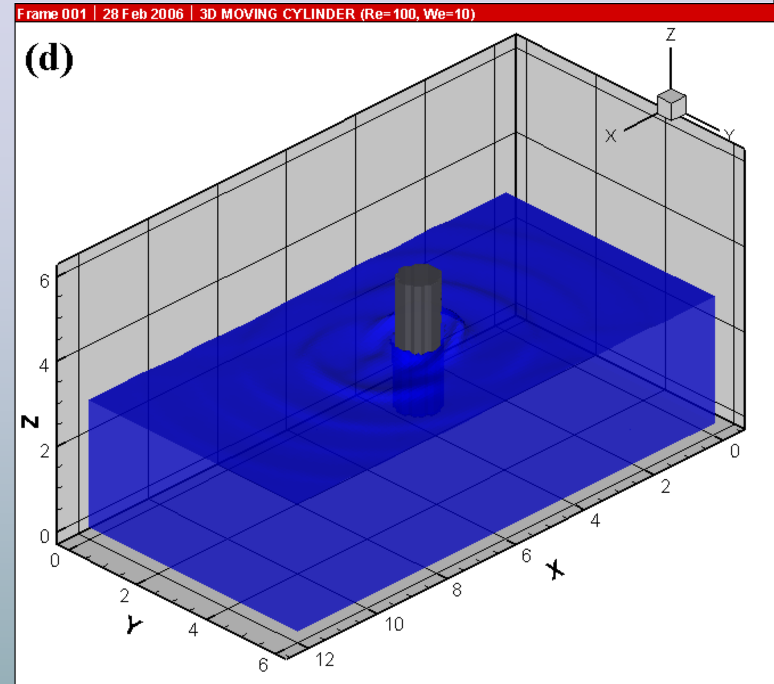
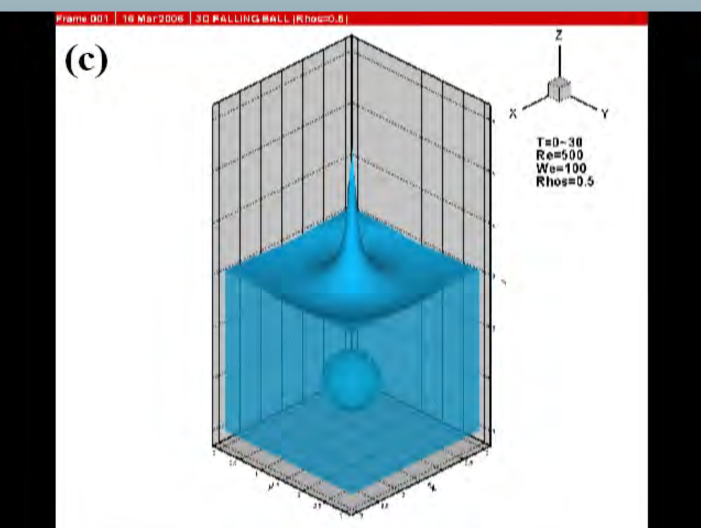
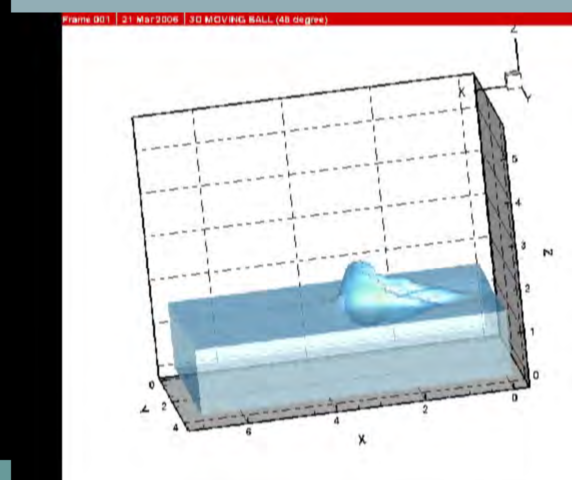
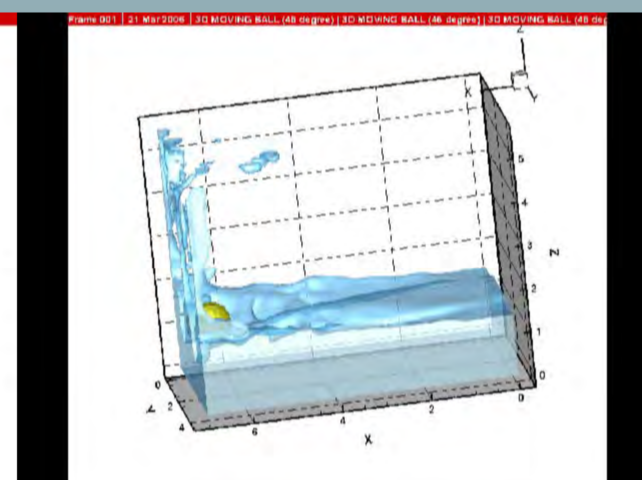










Immersed Boundary Method

The term Immersed Boundary Methods are the class of boundary methods where the calculations are performed on a Cartesian grid that does not conform to the shape of the body in the flow.

The boundary conditions on the body surface are not imposed directly, instead an extra term, called the forcing function/virtual force, is added to the governing equations.

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{u}\mathbf{u}) = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u} + \eta \mathbf{f} + \frac{\mathrm{Gr}}{\mathrm{Re}^2} \theta$$

$$\frac{\partial \theta}{\partial t} + \nabla \cdot (\mathbf{u}\theta) = \frac{1}{\mathrm{RePr}} \nabla^2 \theta + \eta \mathbf{f}_H$$

Simulation with a prescribed motion of a solid object

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\mathbf{uu}) = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\mathbf{u}^* = \mathbf{u}^m + \Delta t \left(\frac{1}{\mathrm{Re}} \nabla^2 \mathbf{u} - \nabla \mathbf{uu} \right)^m \quad \text{(a)}$$

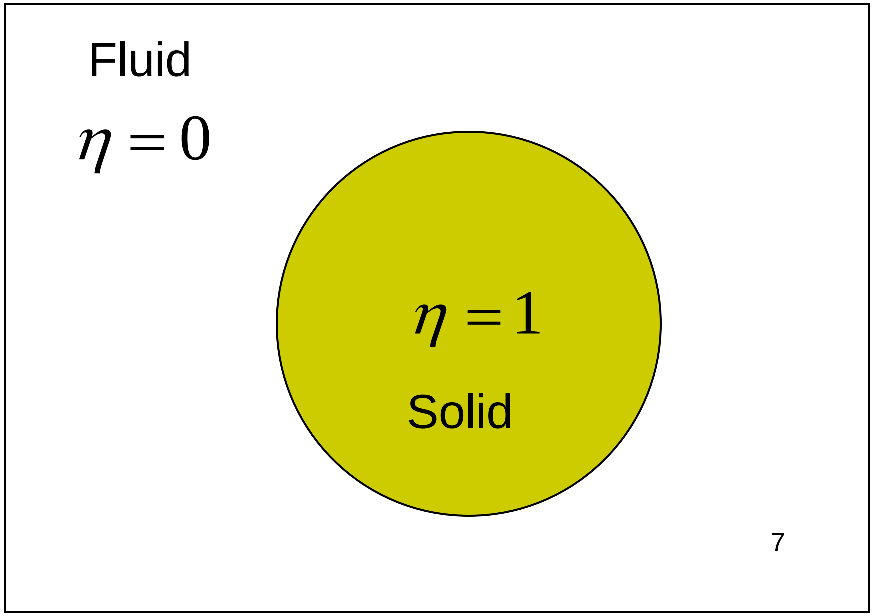
Pressure is solved iteratively by using SOLA algorithm

$$\nabla^2 p^{m+1} = \frac{1}{\Delta t} \nabla \cdot \mathbf{u}^*$$

$$\mathbf{u}^{**} = \mathbf{u}^* - \Delta t \nabla p^{m+1} \quad \text{(b)}$$

$$\mathbf{f}^{m+1} = \eta \frac{\mathbf{u}_s^{m+1} - \mathbf{u}^{**}}{\Delta t} \quad \text{(c)}$$

$$\mathbf{u}^{m+1} = \mathbf{u}^{**} + \Delta t\, \mathbf{f}^{m+1} \quad \text{(d)}$$

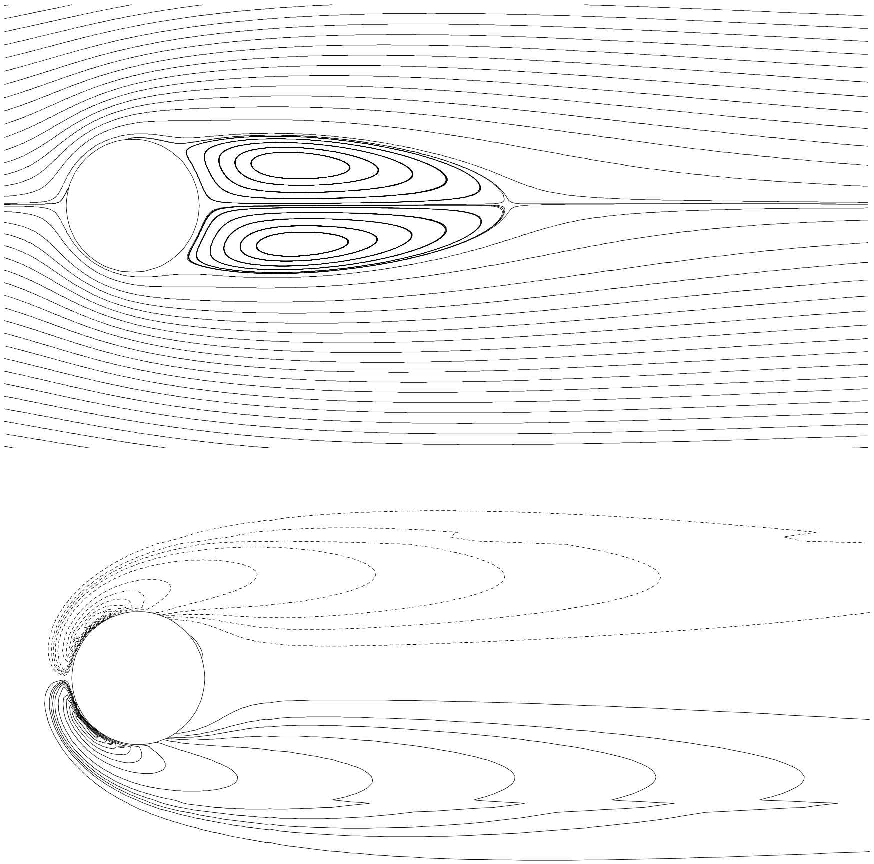


Numerical validations

- Uniform flow past a circular cylinder

(a)

(b)



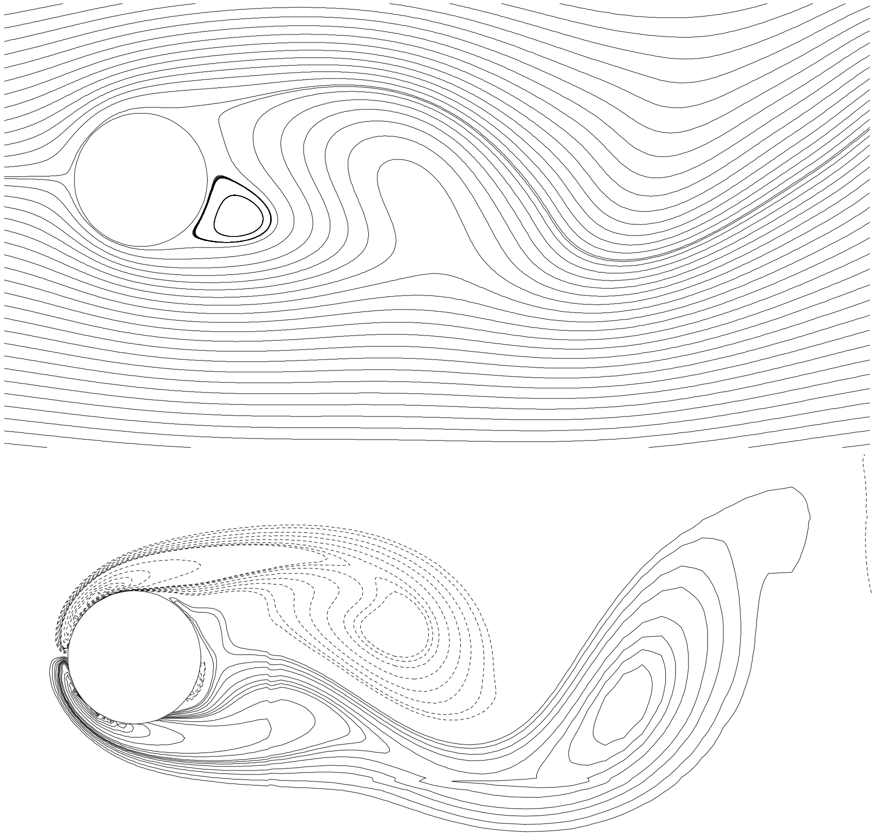
Streamline (a) and vorticity (b) contours at Re = 40

Numerical validations

- Uniform flow past a circular cylinder

(a)

(b)



Streamline (a) and vorticity (b) contours at Re = 100

Table 1. Comparison of Cd, Cl, lw and St at different Re

Authors	Re = 40		Re = 100	
	C_D	l_w	C_D	St
Borthwick [52]	1.507	-	-	-
Tritton [50]	1.480	-	-	-
Chern *et al.* [53]	1.480	2.20	-	-
Dennis and Chang [54]	1.522	2.35	-	-
Su *et al.* [16]	1.630	-	1.40	0.168
Dias and Majumdar [60]	1.540	2.69	1.395	0.171
Pan [24]	1.510	2.18	1.32	0.16
Tseng and Ferziger [20]	1.530	2.21	1.42	0.164
Present study	**1.560**	**2.219**	**1.4**	**0.167**

Simulation for heat transfer

- Conjugate heat transfer case

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \tilde{\alpha} \nabla^2 \theta \quad ; \quad \tilde{\alpha} = \eta \alpha_s + (1 - \eta) \alpha_f$$

- Isothermal case

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \tilde{\alpha} \nabla^2 \theta + \eta \mathbf{f}_H \quad ; \quad \tilde{\alpha} = \eta \alpha_s + (1 - \eta) \alpha_f$$

$$\mathbf{f}_H = \frac{\theta_s^{m+1} - \theta^*}{\Delta t}$$

- Insulating case

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \tilde{\alpha} \nabla^2 \theta \quad ;$$

$$\tilde{\alpha} = \eta \alpha_s + (1 - \eta) \alpha_f \quad ; \quad \alpha_s = 0$$

Simulation for heat transfer

1. Detect the location of boundary cells and determine η

2. Calculate the first intermediate velocity and temperature

$$\frac{\mathbf{u}^{*}-\mathbf{u}^{m}}{\Delta t}=S^{m}+\frac{\mathrm{Gr}}{\mathrm{Re}^{2}}\theta^{m} \qquad \frac{\theta^{*}-\theta^{m}}{\Delta t}=H^{m}$$

where *S* and *H* are the convection and diffusion terms of the momentum and energy equations

$$S^{m}=\frac{3}{2}\left(-\nabla\cdot(\mathbf{uu})+\frac{1}{\mathrm{Re}}\nabla^{2}\mathbf{u}\right)^{m}-\frac{1}{2}\left(-\nabla\cdot(\mathbf{uu})+\frac{1}{\mathrm{Re}}\nabla^{2}\mathbf{u}\right)^{m-1}$$

$$H^{m}=\frac{3}{2}\left(-\nabla\cdot(\mathbf{u}\theta)+\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\nabla^{2}\theta\right)^{m}-\frac{1}{2}\left(-\nabla\cdot(\mathbf{u}\theta)+\frac{1}{\mathrm{Re}\,\mathrm{Pr}}\nabla^{2}\theta\right)^{m-1}$$

Simulation for heat transfer

3. Advance the intermediate velocity

$$\frac{\mathbf{u}^{**}-\mathbf{u}^{*}}{\Delta t}=-\nabla p^{m+1} \qquad \nabla^2 p^{m+1}=\frac{1}{\Delta t}\nabla\cdot\mathbf{u}*$$

4. Calculate virtual force and virtual heat inside solid body

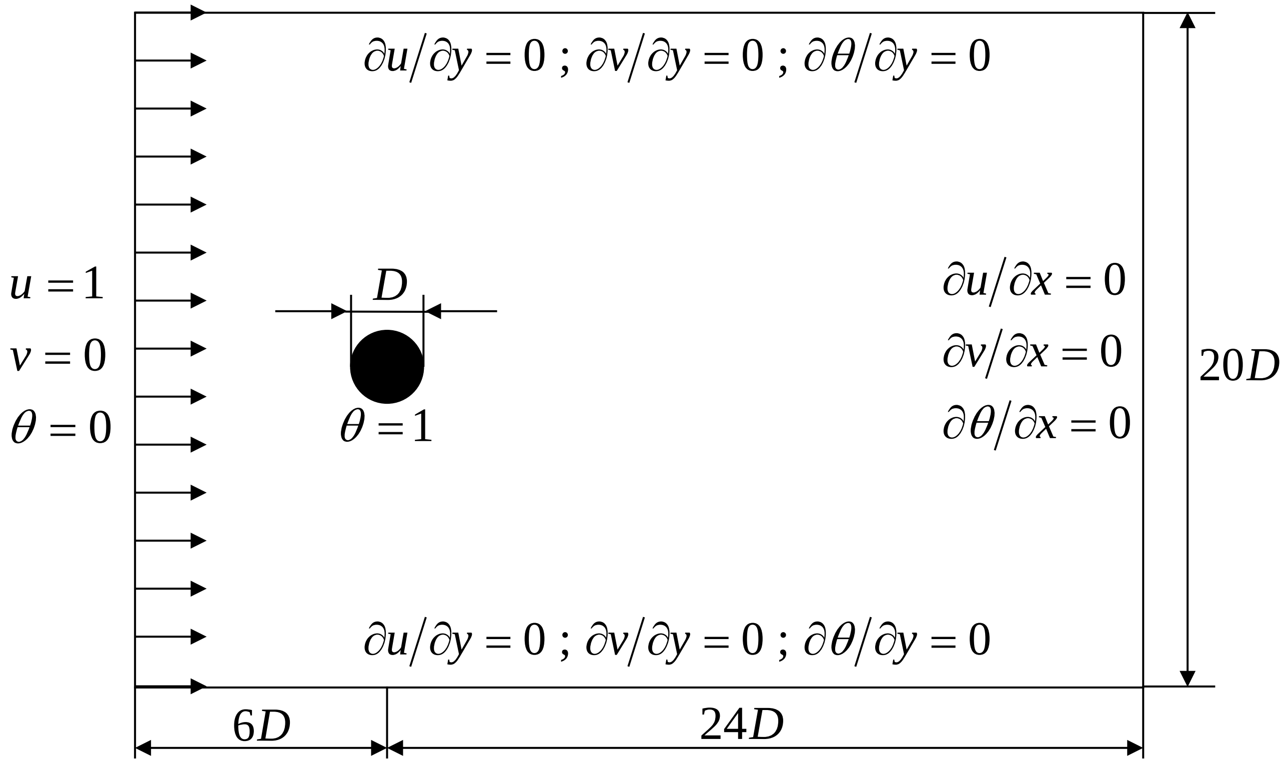
$$\mathbf{f}^{m+1}=\eta\frac{\mathbf{u}_s^{m+1}-\mathbf{u}^{**}}{\Delta t} \qquad \mathbf{f}_H^{m+1}=\eta\frac{\theta_s^{m+1}-\theta^{*}}{\Delta t}$$

5. Update velocity and temperature by imposing calculated virtual force and virtual heat of solid body

$$\frac{\mathbf{u}^{m+1}-\mathbf{u}^{**}}{\Delta t}=\mathbf{f}^{m+1} \qquad \frac{\theta^{m+1}-\theta^{*}}{\Delta t}=\mathbf{f}_H^{m+1}$$

Numerical validations

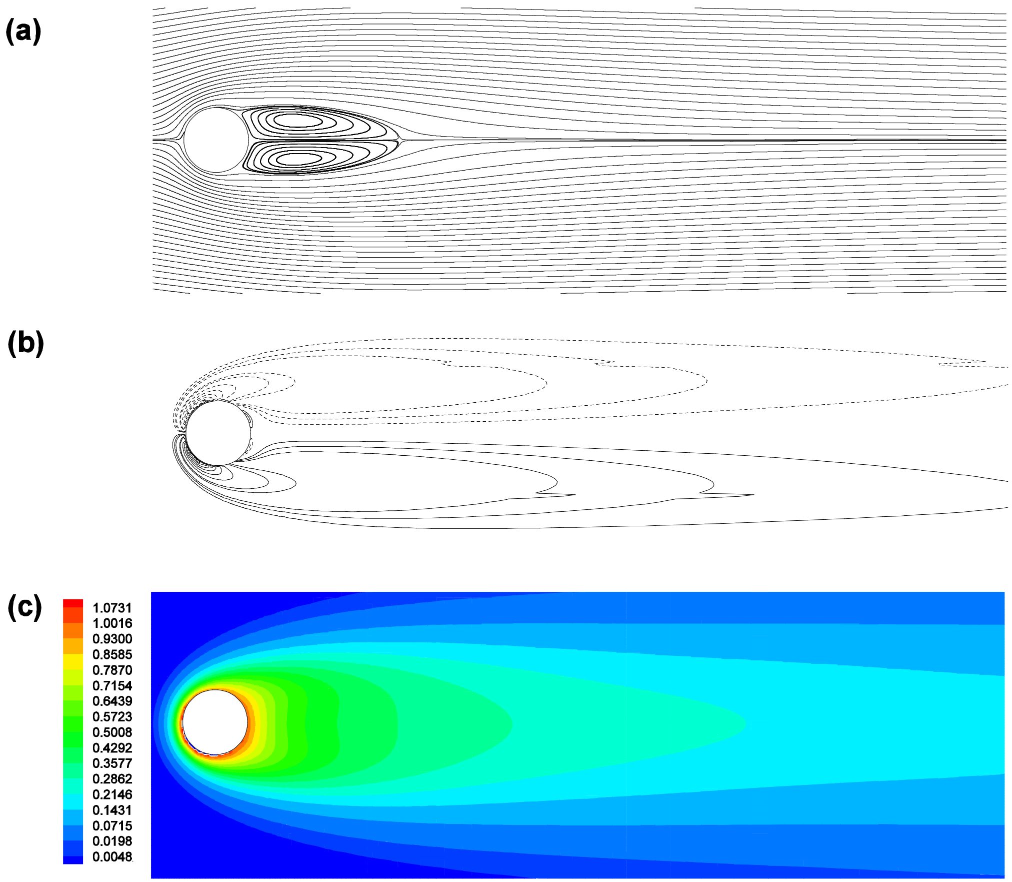
- Forced convection around unbounded heated circular cylinder (Constant temperature cases)



Computational domain and boundary conditions

Numerical validations

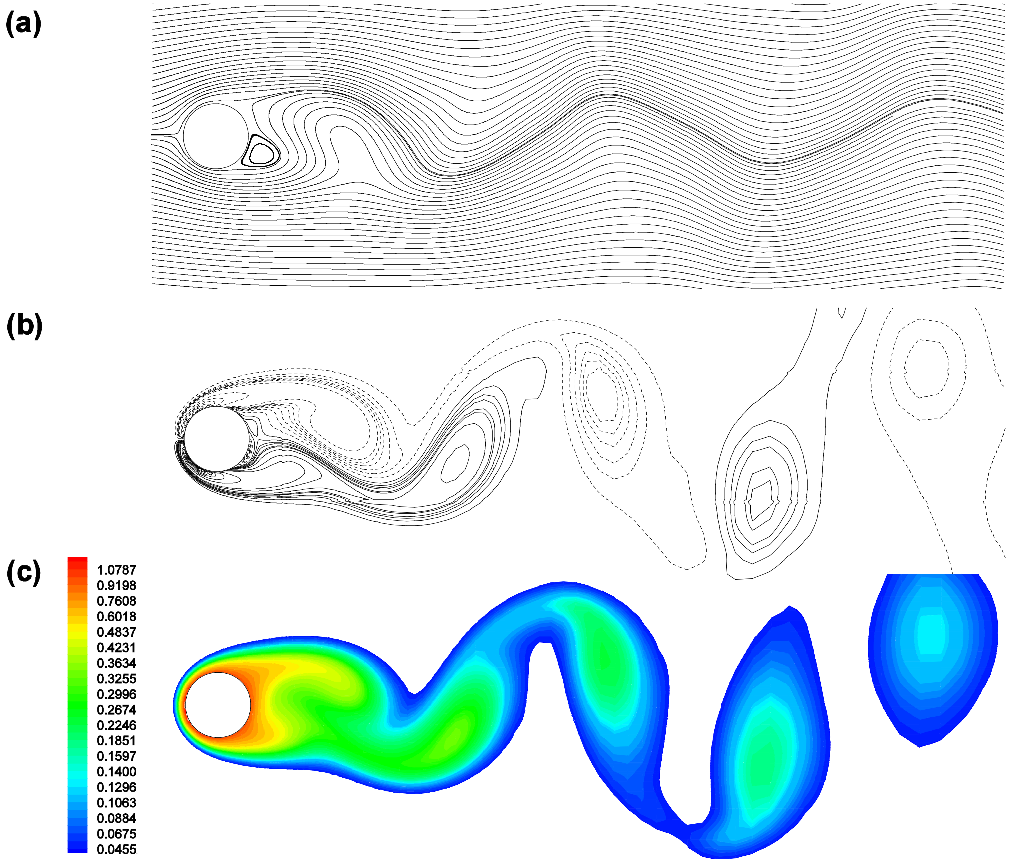
- Forced convection around unbounded heated circular cylinder



(a)Streamline (b) vorticity contours and (c) isotherm contours at Re = 40

Numerical validations

- Forced convection around unbounded heated circular cylinder



(a)Streamline (b) vorticity contours and (c) isotherm contours at Re = 100

Numerical validations

- Forced convection around unbounded heated circular cylinder

Table 1
Comparison of C_D, x_r, St and $\overline{Nu}$ around a circular cylinder placed in an unbounded flow

Authors	Re = 40			Re = 100			
	C_D	x_r	$\overline{Nu}$	$\overline{C_D}$	C_L	St	$\overline{Nu}$
Ecker and Soehngen [15]			3.48				
Ye *et al.* [7]	1.52	2.27					
Vega *et al.* [11]	1.53	2.28	3.62				
Lai and Peskin [4]				1.4473	±0.3229	0.165	
Kim and Choi [15]	1.51			1.33	±0.32	0.165	
Su *et al.* [5]	1.63			1.4		0.168	
Tseng and Ferziger [9]	1.53	2.21		1.42	±0.29	0.164	
Dias and Majumdar [18]	1.54	2.69		1.395	±0.283	0.171	
Pan [12]	1.51	2.18	3.23	1.32	±0.32	0.16	5.02
Present	1.567	2.219	3.32	1.4	±0.322	0.167	5.08

- Natural convection of a heated circular cylinder placed concentrically inside a square enclosure

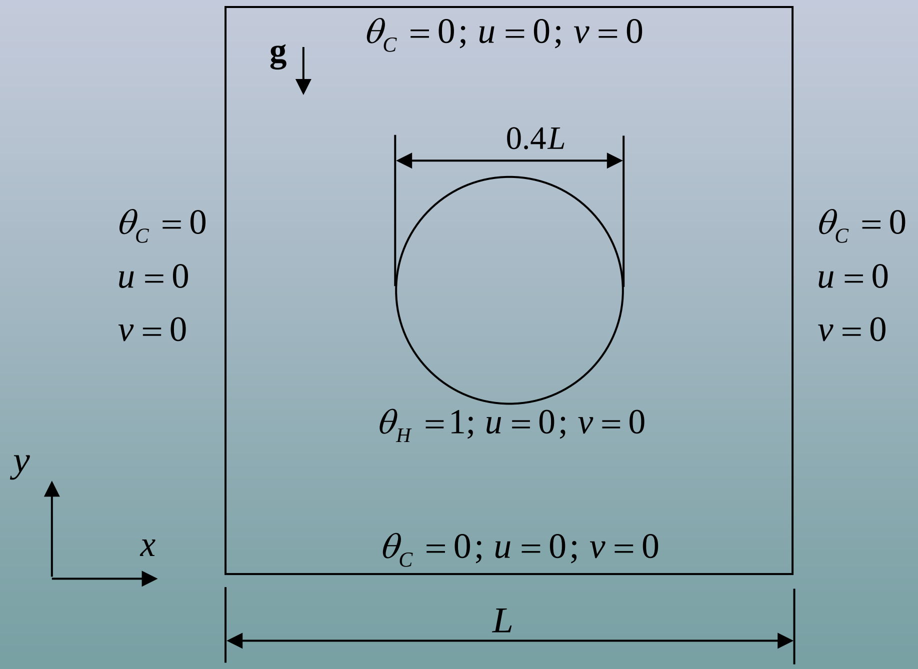
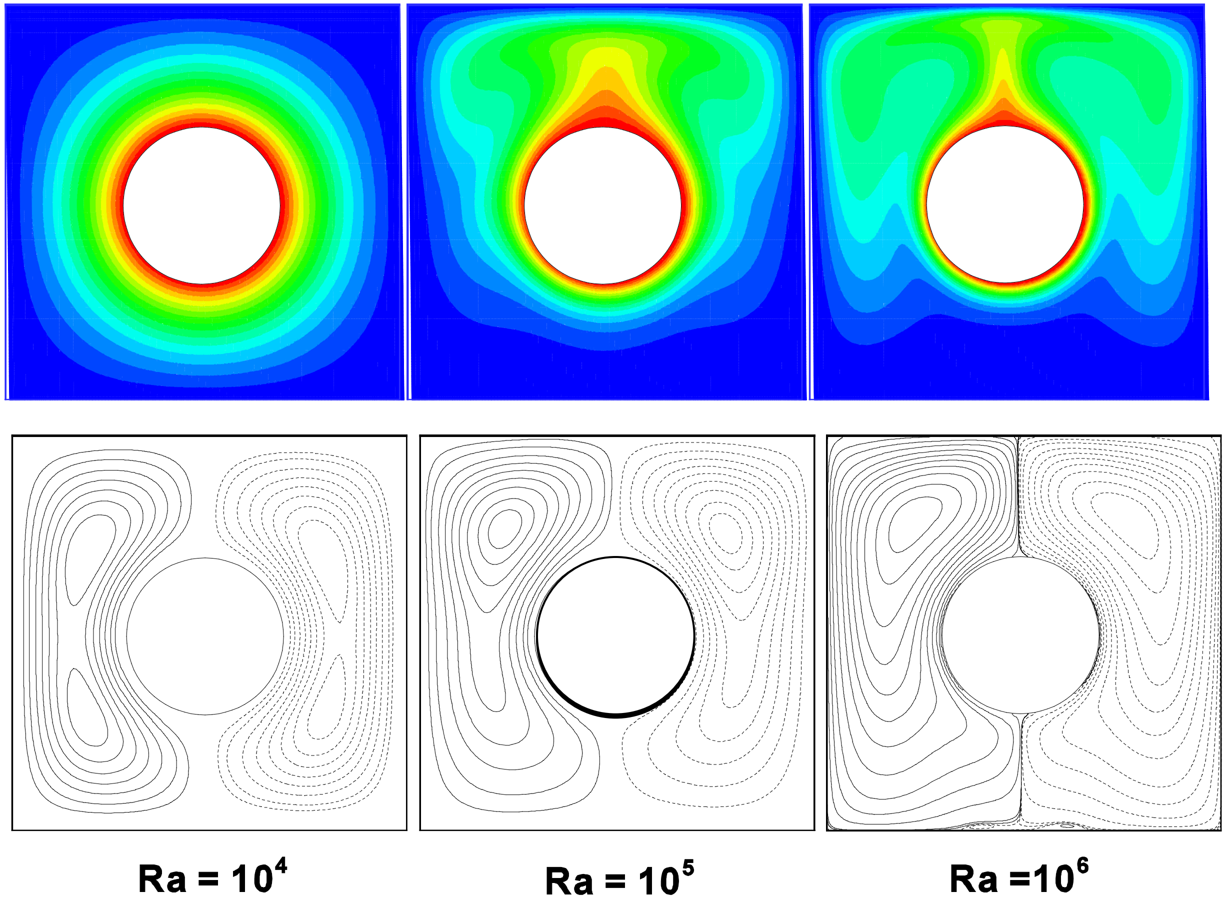


Fig. 6 Computational domain and coordinate system along with boundary conditions

Numerical validations



Natural convection in a square enclosure with a circular cylinder, R = 0.2L

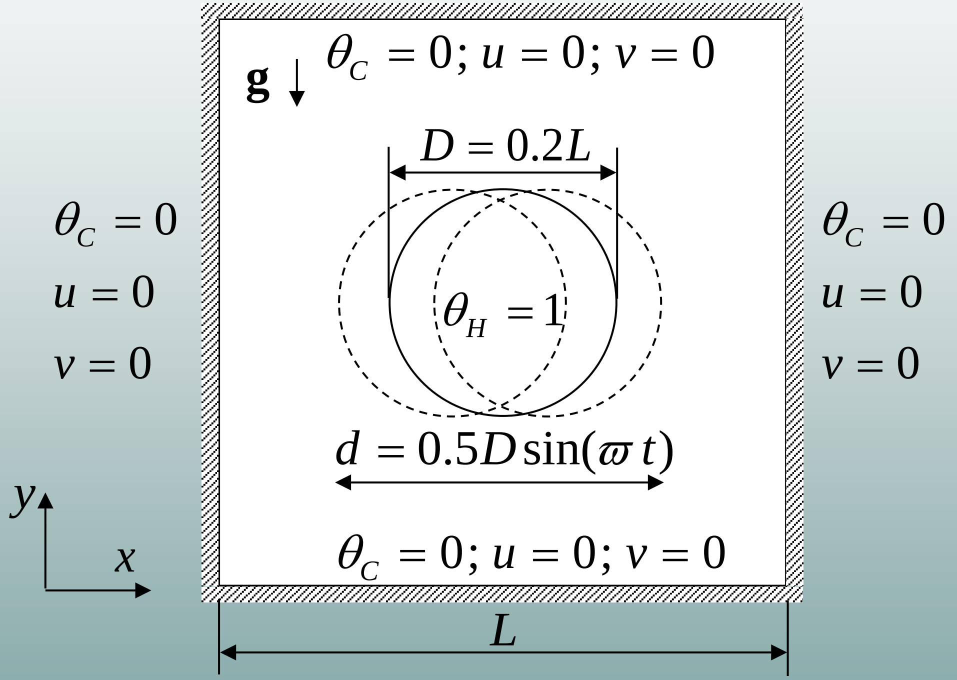
Numerical validations

Table 2

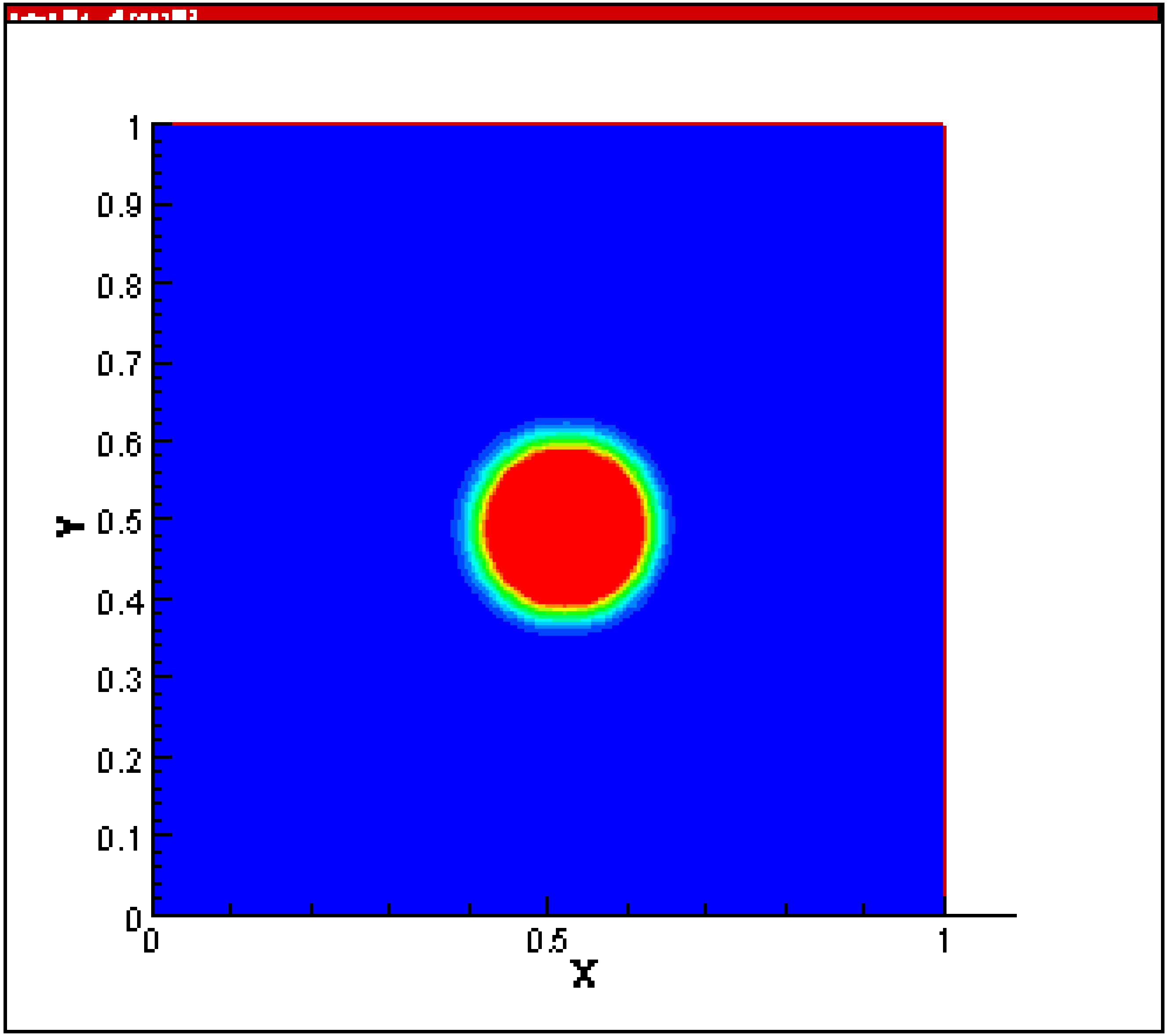
$\overline{\mathrm{Nu}}$ around a circular cylinder placed concentrically inside an enclosure, $D = 0.4L$

Ra	$\overline{\mathrm{Nu}}$			
	Moukalled and Archarya [66]	Shu *et al*. [67]	Kim *et al*. [64]	Present
10^4	3.331	3.245	3.414	3.420
10^5	5.08	4.861	5.1385	5.141
10^6	9.374	8.899	9.39	9.392

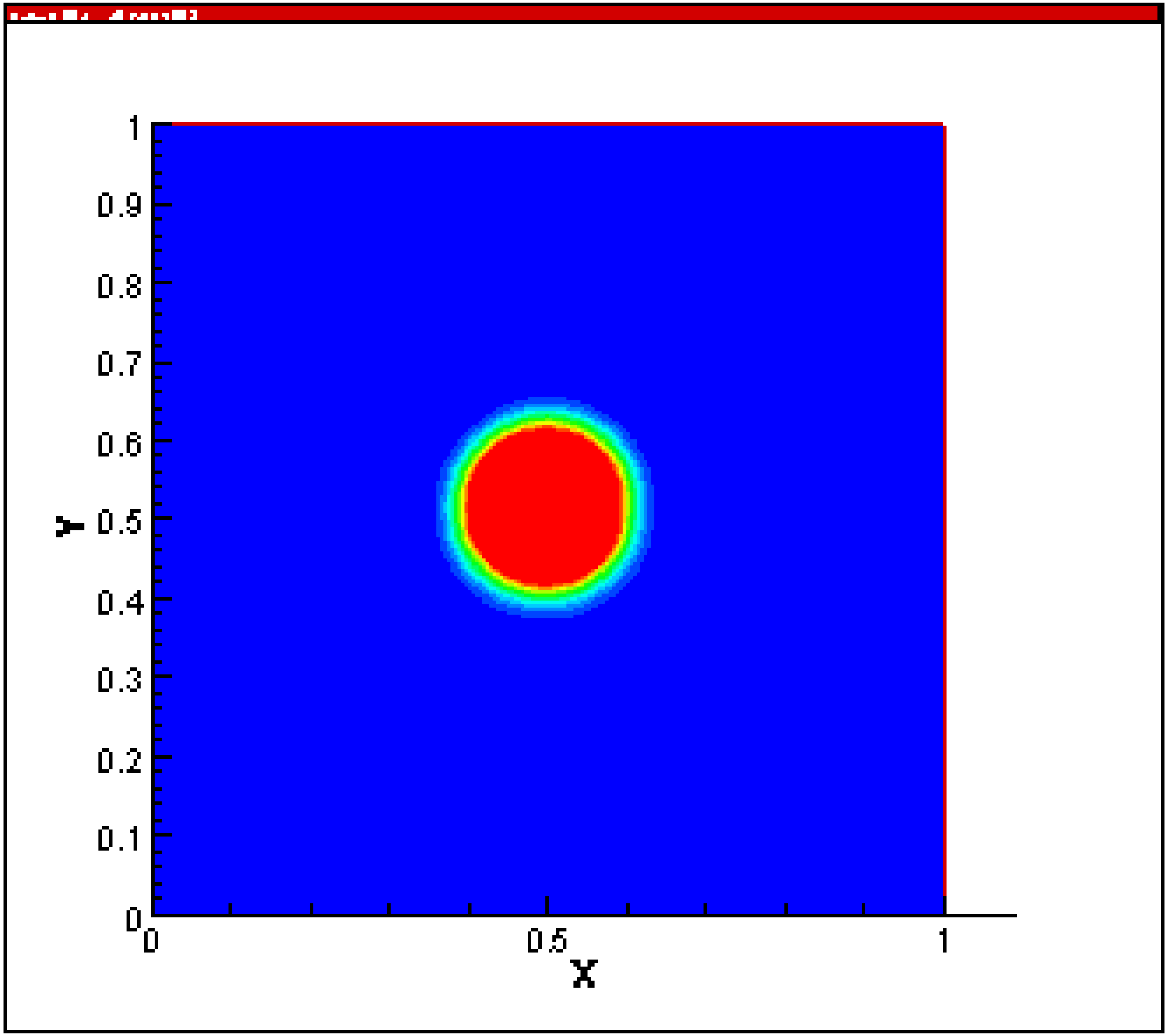
Mixed convection in a square enclosure with a moving heated circular cylinder



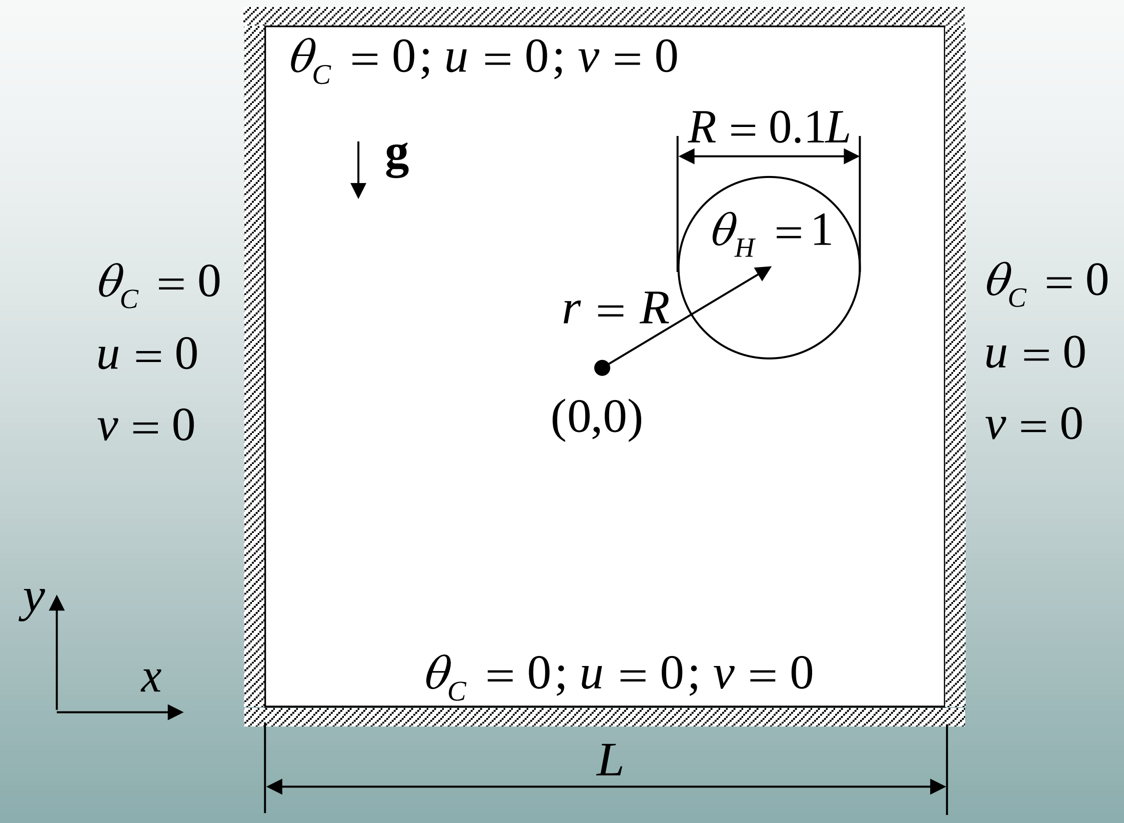
Computational domain and coordinate system along with boundary conditions for mixed-convection in a square enclosure with a horizontally moving circular cylinder



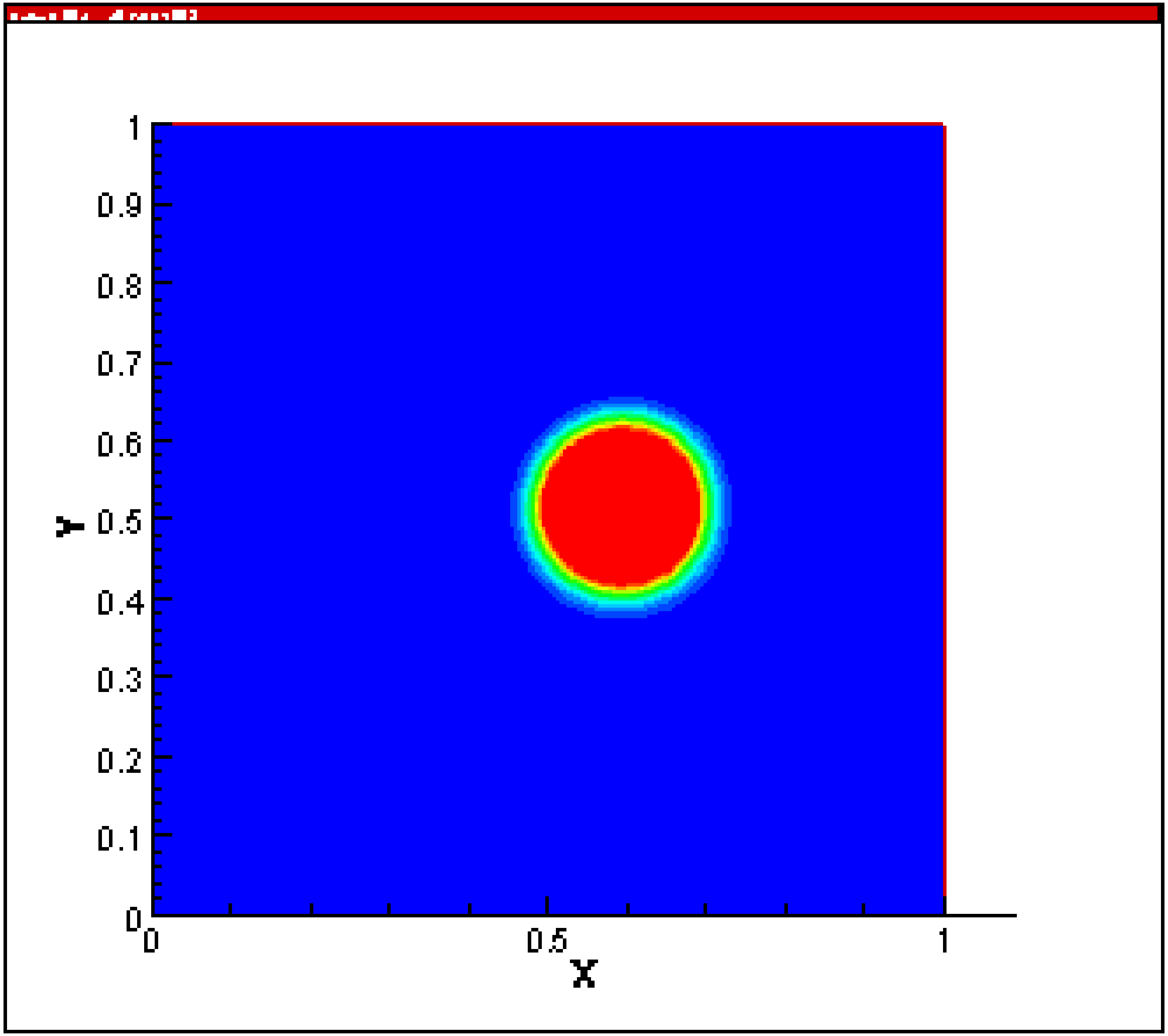
Thermal field during the transient condition for horizontally moving cylinder



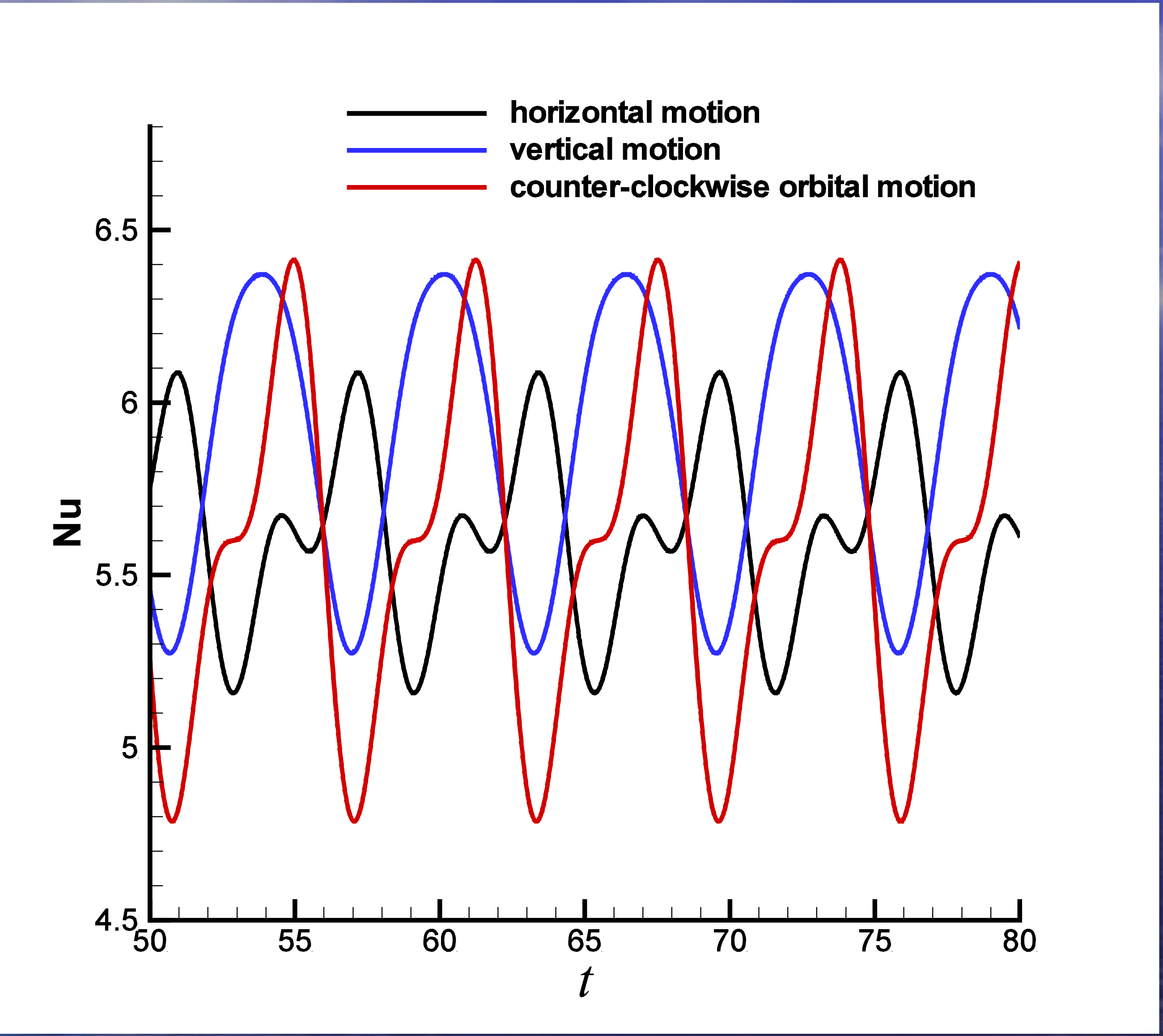
Thermal field during the transient condition for vertically moving cylinder



Computational domain and coordinate system along with boundary conditions for mixed-convection in a square enclosure with a heated circular cylinder moving in ccw

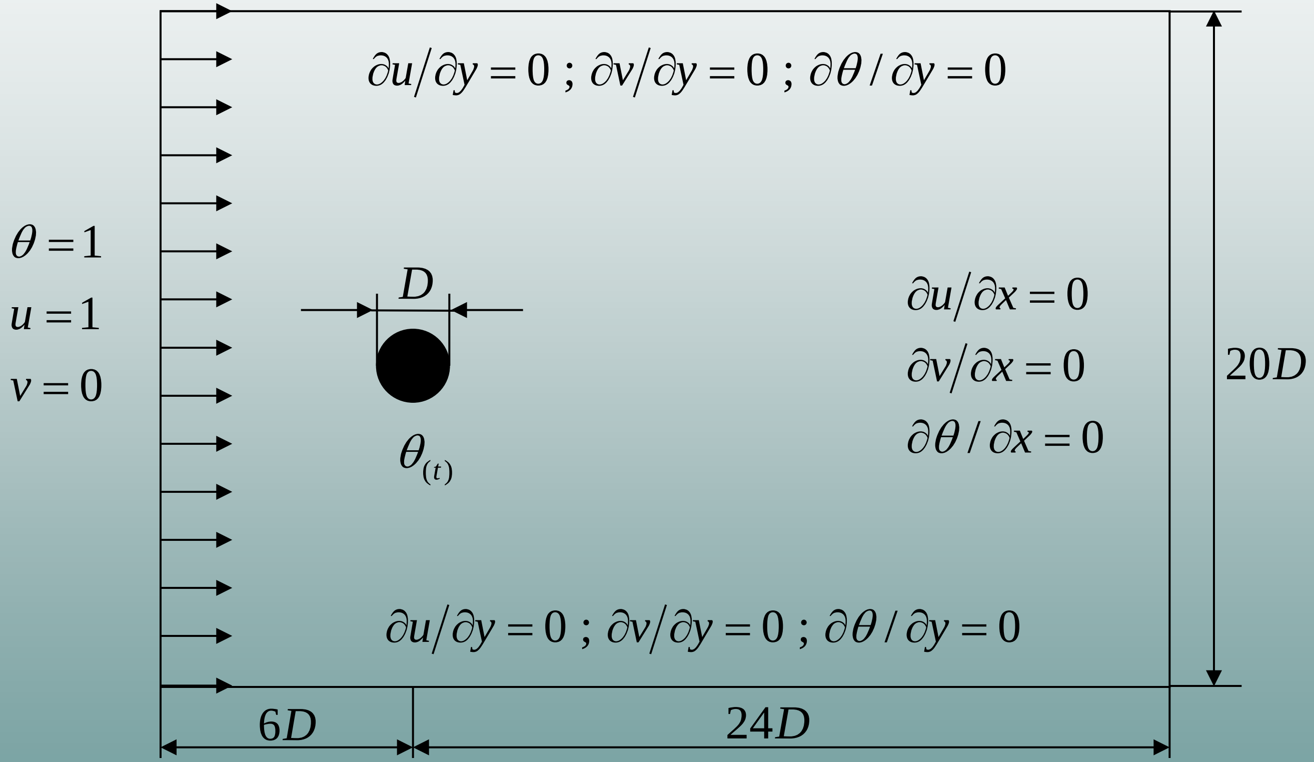


Thermal field during the transient condition for a moving cylinder in orbital motion

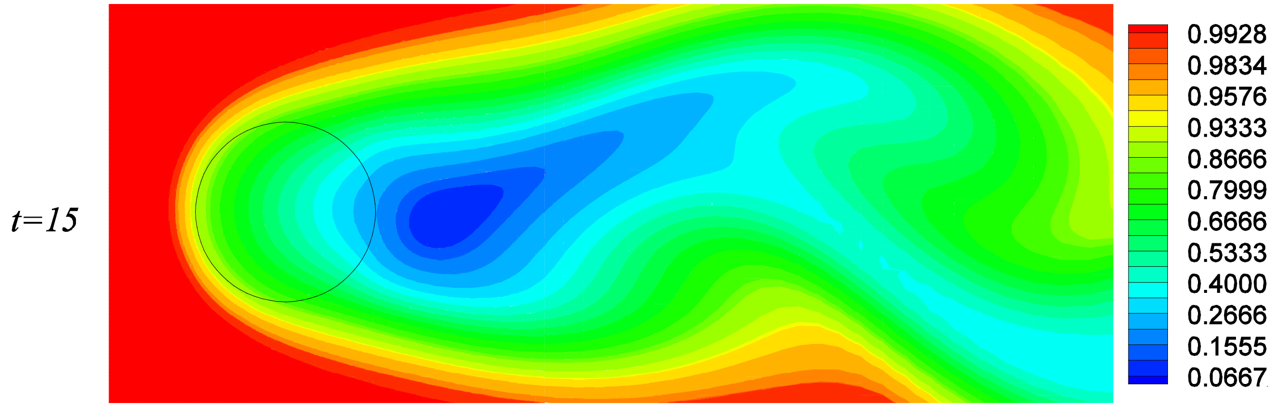


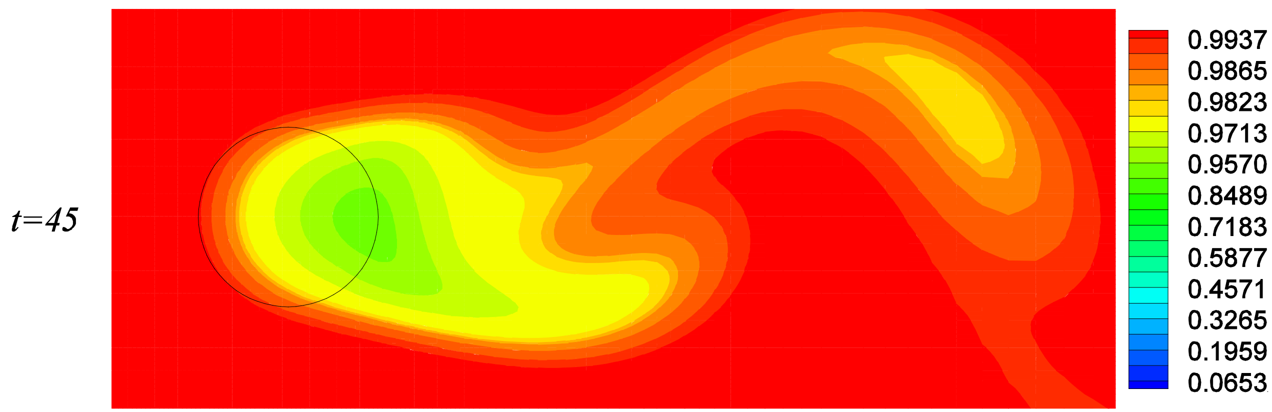
Time history of Nu for a horizontally moving hot cylinder

- Forced convection around a cold circular cylinder (Conjugate heat transfer case)



- Forced convection around a cold circular cylinder (Conjugate heat transfer case)





Isotherm contours at different time.

Conclusions

- Immersed boundary method is a kind of method in computational fluid dynamics where it is not necessary to conform the computational grids to the boundary/shape of an object.
- Cartesian grids are used while the presence of the solid bodies is imposed by means of adequately formulated virtual source terms added to the Navier-Stokes and energy equations.
- Current method handles well fluid-structure interaction with both cases of stationary and moving solid object in computing flow and thermal fields.

Thank you for your attentions.